CSE Qualifying Exam, Fall 2015: Modeling and Simulation Area

Answer 3 out of 4 of the following questions. If you choose to answer all 4, your grade will be based on the questions with the 3 *lowest* scores.

Please write clearly and concisely, explain your reasoning, and show all work.

Question 1. You are tasked with developing a simulation of pedestrian traffic leaving the area around Georgia Tech after a football game. a) Describe a stochastic arrival process you would use to model people leaving one exit of the stadium and entering the street network and explain why you think your model will give credible results. b) Describe what parameters, if any, are needed in your model, and describe how you will determine suitable parameter values. c) Describe in detail how you would validate your model for the arrival process.

Question 2. A repeatable simulation is one where the simulation produces exactly the same results if the simulation is executed over and over again with the same inputs. Consider a *parallel* discrete event simulation program executing on a parallel computer using the Chandy/Misra/Bryant synchronization algorithm. Is such a simulation repeatable? What conditions (if any) must exist for the parallel simulation to be repeatable?

Question 3. Zeroth, first, and second order dead reckoning (DR) models are used in distributed simulations, with higher order models requiring more computational overhead than lower order models with the expectation that they will yield "better" results. Computers are much faster now than when these DR models were invented. Given very fast modern computers, propose a new DR model that even though it will require significantly more computation time, will yield significantly "better" results. Justify your proposal by describing a specific scenario where your DR model will substantially outperform a second order DR model. Explain your metric for evaluating the quality of the DR model.

Question 4. The susceptible-infected-removed (SIR) model is a classic continuous-time conceptual model of the spread of infection in a well-mixed population. In this model, an individual is either *susceptible* to the disease, *infected* with the disease, or *recovered* (or *removed*) from the disease; susceptible individuals may become infected, infected individuals eventually recover, and recovered individuals can no longer become infected. If $S(t)$, $I(t)$, and $R(t)$ are the fraction of such individuals at a given time t, then the SIR model posits that their time-varying relationships obey the system of ordinary differential equations,

$$\frac{dS(t)}{dt} = -\mu S(t) I(t)$$

$$\frac{dI(t)}{dt} = \mu S(t)I(t) - \rho I(t)$$

$$\frac{dR(t)}{dt} = \rho I(t)$$

where μ and ρ are coefficients of *mixing* and *recovery*, and $S(t) + I(t) + R(t) = 1$.

a) In "plain English," explain the form and assumptions of this SIR system, including how to interpret μ and ρ.

b) Suppose you wish to adapt SIR to account for the *social network* that connects individuals. That is, suppose you are given an undirected graph $G = (V, E)$, where $V = \{v_i : 1 \leq i \leq n\}$ represent the population of n individuals, and $E = \{(u, v)\}$ represent the $m = |E|$ connections between individuals u and v. Assume that the presence of an edge (u, v) means the individuals are in contact, whereas the absence of an edge indicates u and v will *never* come in contact. Explain how to adapt the SIR model, given such a graph.

c) Under what conditions will your graph-based SIR model from part b) have a stable equilibrium, as $t \to \infty$?